# Spontaneous Violation of the CP Symmetry in the Higgs Sector of the Next-to-Minimal Supersymmetric Model

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**Abstract**

The spontaneous violation of the CP symmetry in the next-to-minimal supersymmetric standard Model (NMSSM) is investigated. It is found that the spontaneous violation of the CP symmetry can occur in the Higgs sector of the NMSSM for a wide region of the parameter space of the model, at the 1-loop level where the radiative corrections due to the top quark and scalar-top quark loops are found to generate the scalar-pseudoscalar mixings between the two Higgs doublets of the NMSSM. In our model, we assume that the masses of the left-handed and the right-handed scalar-top quarks are not degenerate. And we investigate our model anaytically: We derive analytical formulae of the 1-loop mass matrix for the neutral Higgs bosons. We calculate the upper bound on the lightest neutral Higgs boson mass under the assumption. It is found to be about 140 GeV for our choice of parameter values in the presence of the spontaneous violation of the CP symmetry in the NMSSM. Thus, the possibility of the spontaneous violation of the CP symmetry is not completely ruled out in the Higgs sector of the NMSSM if the masses of the left-handed and the right-handed scalar-top quarks are not degenerate. Further, the phenomenology of the $K$-$\bar{K}$ mixing within the context of our model is studied. The lower bound on CP violating phase in the $K$-$\bar{K}$ mixing is found to increase if either $\tan\beta$ decreases or $A_t$ increases.

# I. INTRODUCTION

The violation of the CP symmetry in the weak interactions has been with us more than several decades since Christenson *et al.* had discovered the CP violating process in weak $K$ decays [1]. In the standard model (SM) of the electroweak interactions [2], the violation of the CP symmetry in the weak interactions is generally explained in terms of the complex phase that exists in the Cabibbo-Kobayashi-Maskawa (CKM) matrix for the charged weak current [3]. However, if the nature can be described by other theories than the SM, one may consider other possibilities of the CP violation than the complex phase in the CKM matrix. One of the possibilities is that the CP symmetry is spontaneously violated in the Higgs sector. If the violation of the CP symmetry be occurred in this way, it is required that the relevant models should necessarily have at least two Higgs doublets [4].

Evidently, we know that there are such models which have two Higgs doublets: The supersymmetric extensions of the standard models. They are the most strong candidates for the fundamental theory of the nature beyond the SM, which embrace many important characteristics of the SM. As is well known, the supersymmetric extensions of the standard model need at least two Higgs doublets, in order to give masses to the up-quark sector and the down-quark sector separately [5]. Therefore, in those supersymmetric models, the CP symmetry can be violated spontaneously in their Higgs sectors, in principle.

Naturally, many authors have investigated the minimal version of the supersymmetric standard model (MSSM) [6]. They have found that, at the tree-level, the vacuum cannot spontaneously violate the CP symmetry. It is because certain restrictions from the supersymmetry are imposed on the tree-level Higgs sector of the MSSM to conserve the CP symmetry. Thus, radiative corrections are taken into account in order to see if the CP symmetry be violated spontaneously in the MSSM at higher level. A couple of reasonable scenarios have been proposed in which the radiatively corrected Higgs potential of the MSSM leads actually the spontaneous violation of the CP symmetry. An unacceptable side effect of the spontaneous CP violation scenario in the radiatively corrected Higgs potential of the MSSM is that it leads to a very light neutral Higgs boson which has already been ruled out by the Higgs search at LEP1. Consequently, in order to obtain an experimentally viable spontaneous violation of the CP symmetry scenario in the SUSY model, the Higgs sector of the MSSM has to be extended.

Among various non-minimal supersymmetric models the simplest one is the next-to-minimal supersymmetric standard model (NMSSM), in which a new neutral Higgs singlet field is introduced to the already-existing two Higgs doublet fields of the MSSM [7]. The NMSSM is apparently appropriate because, at the tree level, the upper bound on the lightest neutral Higgs boson mass of the NMSSM is larger than that of the MSSM. Moreover, in the NMSSM, the superpotential contains a new dimensionless coupling coefficient for the cubic interaction between the two Higgs doublet fields and the Higgs singlet field, which may replace the parameter $\mu$ in the superpotential of the MSSM by developing the vacuum expectation value of the singlet Higgs field of the NMSSM.

However, it has been shown that the NMSSM can not produce the violation of the CP symmetry, at least at the tree level by Romão [8]. It is because the vacuum which are chosen to violate the CP symmetry is found to have a mode with a negative squared masses for the neutral Higgs bosons. Consequently, the inclusion of the higher-order corrections to the NMSSM Higgs potential have been considered as a next step.

Recently, Babu and Barr [9] have analyzed in the NMSSM at the 1-loop level to include the radiative corrections if they contribute to the spontaneous violation of the CP symmetry. It has been assumed in their analysis that the mass of the left-handed scalar-top quark is degenerate with that of the right-handed one. Their analysis show that the mass of the charged Higgs boson does not change by taking into account these radiative corrections and its mass is estimated to be smaller than about 110 GeV. Moreover, there is no relative phase among the vacuum expectation values in the 1-loop effective potential including these radiative corrections. In this case, the CP-violating minima are possible only when $\lambda$ is very small for $\tan\beta = 1$, and thus two neutral Higgs bosons become very light.

Independently, Haba, Matsuda, and Tanimoto [10] have investigated the spontaneous violation of the CP symmetry in the Higgs sector of the NMSSM using the 1-loop effective potential including radiative corrections due to top quark, scalar-top quark, bottom quark, and scalar-bottom quark contributions. Here, the degeneracy in the left- and the right-handed components have not been assumed. They have found numerically that the spontaneous violation of the CP symmetry can occur only in a very restricted region of the parameter space. For $\lambda = 0.16$ and $\tan\beta = 1$, their numerical calculations have estimated the upper bound on the lightest neutral Higgs boson mass to be about 36 GeV, and the sum of two lightest Higgs boson masses is around 93 GeV. Haba *et al.* also have calculated the mass of the charged Higgs boson. It has been found to be dependent crucially on the soft SUSY breaking scalar-quark mass $m_Q$. It is noticeable that the mass of the chasrged Higgs boson may be as large as about 721 GeV, which is considerably different from the case of Babu and Barr.

In this paper we are going a little further into the investigation of the spontaneous violation of the CP symmetry in the neutral Higgs sector of the NMSSM. We also assume no degeneracy between the left-handed and the right-handed components of the scalar quarks, and we derive the 1-loop corrections as far as we can analytically, in order to investigate the effects of parameters of the NMSSM upon the spontaneous violation of the CP symmetry.

At initial stage, our Lagrangian density is assumed too to be invariant with respect to CP property. After electroweak symmetry breaking, the CP symmetry is spontaneously broken in the vacuum state of the potential. Our assumptions go in parallel with the above two analyses [9, 10] that the vacuum expectation values of the two Higgs doublet fields as well as that of the Higgs singlet field are complex in the Higgs sector of the NMSSM. Naturally, one of the three CP-violating complex phases can be eliminated by redefining the Higgs fields. Thus, the spontaneous violation of the CP symmetry is generated in terms of the remaining two physical phases.

The radiative corrections to the neutral Higgs boson masses are investigated by using the effective potential method [11]. The 1-loop effective potential contains the contribution of the top quark and scalar-top quark contributions. The difference bewteen the work of Babu and Barr and ours is that we assume no degeneracy between the left- and the right-handed components of the scalar quarks, as aforementioned.

Since there is an additional Higgs singlet field in the NMSSM, the mass matrix for neutral Higgs boson is a $5 \times 5$ matrix. We derive an analytical formula for the neutral Higgs boson mass matrix at the 1-loop level. This is the difference from the works of Haba *et al.*. Even though at the tree level the scalar-pseudoscalar mixing elements between two Higgs doublets are explicitly zero, radiative corrections generate non-zero values for these elements. The real symmetric mass matrix can be diagonalized by an orthogonal matrix and then the NMSSM leads to five neutral

Higgs bosons. In the spontaneous violation of the CP symmetry scenario, the theoretical upper bound on the lightest neutral Higgs boson mass of the NMSSM is calculated in the context of the effective potential formalism.

Then, we apply our result to the CP violations in $K$-$\bar{K}$ mixing numerically. Within the parameter space of the NMSSM that we consider, the CP violation in $K$-$\bar{K}$ mixing are studied within the framework of the NMSSM. We find that even though the scalar Higgs bosons have not been discovered yet at LEP2, the possibility of the spontaneous violation of the CP symmetry is not completely ruled out in the Higgs sector of the NMSSM.

This paper is organized as follows. In the next section we breifly describe the Higgs sector of the NMSSM within the scenario of the spontaneous violation of the CP symmetry in the Higgs sector of the NMSSM at the tree level, and then at the 1-loop level in the third section. The radiative corrections to the tree level Higgs potential are then taken into account to generate a viable CP-violating vacuum in the Higgs sector. The mixings between two Higgs doublets can occur through the radiative corrections due to the contributions from the top quark and the scalar-top quark contributions. The exact analytical expressions of the elements of the neutral Higgs boson mass matrix are derived in our scenario. At the 1-loop level, the parameter space of the NMSSM, and hence masses of the scalar Higgs bosons are constrained by the negative experimental results from LEP1. The upper bound on the lightest neutral Higgs boson mass is calculated to be about 140 GeV for our choice parameter values in the presence of the spontaneous violation of the CP symmetry in the NMSSM. In the fourth section, we apply our model into the case of the CP violation in the $K$-$\bar{K}$ mixing within the context of the spontaneous violation scenario in the NMSSM Higgs sector. The conclusions are given in the last section.

## II. NEUTRAL HIGGS SECTOR AT TREE LEVEL

Here in this section, we briefly review the concept of the spontaneous violation of the CP symmetry in the NMSSM at the tree level. As is well known, the Higgs sector of the NMSSM consists of two Higgs doublet superfields $H_1^T = (H_1^0,\ H_1^-)$ and $H_2^T = (H_2^+,\ H_2^0)$, plus a Higgs singlet superfield $N$. The superpotential of the NMSSM contains only terms with dimensionless couplings. Ignoring all quark and lepton Yukawa couplings except for that of the top quark, the relevant part for the superpotential can be written as

$$W = h_t Q H_2 t_R^c + \lambda N H_1 H_2 - \frac{k}{3} N^3 \ ,$$

where we denote for simplicity that $H_1 H_2 = H_1 \epsilon H_2 = H_1^0 H_2^0 - H_1^- H_2^+$. The superfield $Q^T = (t_L,\ b_L)$ consists of the left-handed quarks of the third generation and the superfield $t_R^c$ denotes the charge conjugate of the right-handed top quark.

In the above superpotential, a global U(1) Peccei-Quinn symmetry is explicitly broken by the presence of the cubic term in $N$. Not for the term, i.e., if $k = 0$, the Peccei-Qiunn symmetry would persist in the NMSSM superpotential, and the tree-level Higgs potential would lead to a massless pseudoscalar Higgs boson, or the pseudo-Goldstone boson, after electroweak gauge symmetry breaking.

The tree level Higgs potential in the NMSSM can be written in terms of $F$-terms, $D$-terms, and the soft SUSY breaking terms as

$$V_{\rm tree} = V_F + V_D + V_{\rm soft}$$

where

$$\begin{aligned}
V_F &= |\lambda|^2[(|H_1|^2 + |H_2|^2)|N|^2 + |H_1 H_2|^2] + |k|^2|N|^4 - (\lambda k^* H_1 H_2 N^{*2} + {\rm H.c.}) \ , \\
V_D &= \frac{1}{8}(g_1^2 + g_2^2)(|H_2|^2 - |H_1|^2)^2 \ , \\
V_{\rm soft} &= m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + m_N^2 |N|^2 - (\lambda A_\lambda H_1 H_2 N + \frac{1}{3} k A_k N^3 + {\rm H.c.}) \ ,
\end{aligned} \tag{1}$$

with $g_1$ and $g_2$ being the U(1) and SU(2) gauge coupling constants, respectively.

We assume that there is no explicit violation of the CP symmetry in the Higgs sector [12]. Therefore, the parameters $\lambda$, $k$, $A_\lambda$, and $A_k$ are assumed to be all real. On the contrary, we assume that the violation of the CP symmetry can occur spontaneously through the existence of the complex phases in the vacuum expectation values of the three neutral Higgs fields:

$$\begin{aligned}
< 0|H_1^0|0 > &= v_1 e^{i\varphi_1} \ , \\
< 0|H_2^0|0 > &= v_2 e^{i\varphi_2} \ , \\
< 0|N|0 > &= x e^{i\varphi_3} \ ,
\end{aligned} \tag{2}$$

where $v_1$, $v_2$, and $x$ are assumed to be positive, and three non-trivial phases are introduced into the vacuum expectation values. As usual, $\tan\beta$ is defined as $v_2/v_1$, and the top quark mass is generated by $v_2$ of the Higgs doublet $H_2$ as $m_t = h_t v_2$. One of the three complex phases can be eliminated by redefining the Higgs fields. The remaining two physical phases may be chosen as [10]

$$\begin{aligned}
\theta &= \varphi_1 + \varphi_2 + \varphi_3 \ , \\
\delta &= 3\varphi_3 \ .
\end{aligned} \tag{3}$$

In the spontaneous violation scenario of the CP symmetry, the vacuum is defined as the stationary point with respect to the two CP violating phases $\theta$ and $\delta$. This stationary point or the minimum point is the CP violating vacuum. Thus, the CP violating vacuum with respect to these phases satisfies two minimum equations

$$\begin{aligned}
&\frac{\partial < V_{\rm tree}(v_1, v_2, v_3, \theta, \delta) >}{\partial \delta} = 0 \ , \\
&\frac{\partial < V_{\rm tree}(v_1, v_2, v_3, \theta, \delta) >}{\partial \theta} = 0 \ .
\end{aligned} \tag{4}$$

Furthermore, one can use the three minimum conditions for $v_1$, $v_2$, and $v_3$ to eliminate the soft supersymmetry-breaking masses $m_{H_1}^2$, $m_{H_2}^2$, and $m_N^2$ in the Higgs potential.

Now, in terms of those vacuum expectation values, the three neutral Higgs fields can be rewritten by shifting them around the CP violating vacuum. They have three scalar components and three pseudoscalar components: One of the mass eigenstates of the three pseudoscalar components is a massless Goldstone mode. This Goldstone mode can be gauged away by a unitary gauge tansformation, and we are left with five components for the neutral Higgs fields. Consequently, the three neutral Higgs fields may be expressed in terms of the five components as

$$
\begin{aligned}
H_1^0 &= e^{i\varphi_1}\left\{v_1 + \frac{1}{\sqrt{2}}(S_1 + i\sin\beta P)\right\} , \\
H_2^0 &= e^{i\varphi_2}\left\{v_2 + \frac{1}{\sqrt{2}}(S_2 + i\cos\beta P)\right\} , \\
N &= e^{i\varphi_3}\left\{x + \frac{1}{\sqrt{2}}(X + iY)\right\} ,
\end{aligned}
\tag{5}
$$

where $S_1$, $S_2$, and $X$ are the scalar components, and $P$ and $Y$ are the pseudoscalar components.

The mass matrix for the five neutral Higgs bosons is obtained by the second derivatives of the Higgs potential with respect to the corresponding Higgs fields evaluated at the CP violating vacuum as a symmetric $5 \times 5$ matrix:

$$
M_{ij}^2 = M_{ji}^2 = \left[\begin{array}{cc} M_{S_1,\ S_2,\ X}^{S_1,\ S_2\ X} & M_{S_1,\ S_2,\ X}^{A,\ Y} \\ & \\ (M_{S_1,\ S_2,\ X}^{A,\ Y})^T & M_{A,\ Y}^{A,Y} \end{array}\right] ,
$$

where, respectively, the upper-left $3 \times 3$ submatrix and the lower-right $2 \times 2$ submatrix correspond to the scalar part and pseudoscalar part. The upper-right $3 \times 2$ as well as the lower-left submatrices correspond to the scalar-pseudoscalar mixing part in the nutral Higgs boson matrix. As aforementioned, if the CP symmetry is conserved in the Higgs sector, the submatrix for the scalar-pseudoscalar mixing would not exist in the neutral Higgs boson mass matrix.

We obtain the tree-level elements for the mass matrix of the five neutral (scalar and pseudoscalar) Higgs bosons explicitly as

$$
\begin{aligned}
M_{11}^2 &= (m_Z\cos\beta)^2 + \lambda x(A_\lambda\cos\theta + kx\cos(\theta-\delta))\tan\beta , \\
M_{22}^2 &= (m_Z\sin\beta)^2 + \lambda x(A_\lambda\cos\theta + kx\cos(\theta-\delta))\cot\beta , \\
M_{33}^2 &= (2kx)^2 - kxA_k\cos\delta + \frac{\lambda}{2x}v^2 A_\lambda\sin 2\beta\cos\theta , \\
M_{44}^2 &= \frac{2\lambda x(A_\lambda\cos\theta + kx\cos(\theta-\delta))}{\sin 2\beta} , \\
M_{55}^2 &= \frac{\lambda v^2}{2x}A_\lambda\sin 2\beta\cos\theta + 3kxA_k\cos\delta + 2\lambda k v^2\sin 2\beta\cos(\theta-\delta) , \\
M_{12}^2 &= (\lambda^2 v^2 - \frac{1}{2}m_Z^2)\sin 2\beta - \lambda x(A_\lambda\cos\theta + kx\cos(\theta-\delta)) , \\
M_{13}^2 &= 2\lambda^2 xv\cos\beta - \lambda v\sin\beta(A_\lambda\cos\theta + 2kx\cos(\theta-\delta)) , \\
M_{14}^2 &= 0 , \\
M_{15}^2 &= -3\lambda kvx\sin\beta\sin(\theta-\delta) , \\
M_{23}^2 &= 2\lambda^2 xv\sin\beta - \lambda v\cos\beta(A_\lambda\cos\theta + 2kx\cos(\theta-\delta)) ,
\end{aligned}
$$

$$
\begin{aligned}
M_{24}^2 &= 0 \ , \\
M_{25}^2 &= -3\lambda k v x \cos\beta \sin(\theta - \delta) \ , \\
M_{34}^2 &= \lambda k v x \sin(\theta - \delta) \ , \\
M_{35}^2 &= 2\lambda k v^2 \sin 2\beta \sin(\theta - \delta) \ , \\
M_{45}^2 &= \lambda v (A_\lambda \cos\theta - 2kx \cos(\theta - \delta)) \ .
\end{aligned} \tag{6}
$$

Here, one should note that $M_{14}^2 = M_{24}^2 = 0$ implies that there are no scalar-pseudoscalar mixings between the two Higgs doublets (between $H_1$ and $H_2$) in the Higgs sector. That is, the spontaneous violation of the CP symmetry is induced by the presence of the scalar-pseudoscalar mixing terms between among Higgs doublets and the Higgs singlet (between $H_1 H_2$ and $N$) and the cubic term of the singlet itself (among $N^3$) at the tree level.

It can be seen that all the scalar-pseudoscalar mixings vanish if $\theta = \delta$; in this case, at the tree level, the spontaneous violation of the CP symmetry can not occur in the Higgs sector. Then, the five neutral Higgs bosons are decomposed as three scalar Higgs bosons and two pseudoscalar Higgs bosons, and there is no mixing between the scalar and the pseudoscalar Higgs bosons: The $5 \times 5$ mass matrix is decomposed as a $3 \times 3$ and a $2 \times 2$ submatrices. Therefore, in the NMSSM, the spontaneous violation of the CP symmetry can be realized by the presence of two phases $\theta$ and $\delta$.

However, assuming that the two CP violation phases are not equal, the scalar-pseudoscalar mixing may happen inevitably in the Higgs sector, but it has been observed that in large areas of the parameter space of the NMSSM the spontaneous violation does not occur at the tree level, because one can always find a mode with a negative mass squared at the CP violating vacuum of the Higgs potential [8].

## III. NEUTRAL HIGGS SECTOR AT 1-LOOP LEVEL

Now, we turn to the 1-loop level. Since the radiative corrections due to the top quark and scalar-top quark contributions give significant contributions to the tree level Higgs boson masses, we include these contributions in order to see their effects on the spontaneous violation of the CP symmetry. The full Higgs potential is composed of the tree level part and the 1-loop level part written as

$$
V = V_{\rm tree} + V_{\rm 1-loop} \ .
$$

According to the Coleman-Weinberg mechanism [14], the 1-loop effective potential including the contributions of the top quark and scalar-top quark loops is given by

$$
V_{\rm 1-loop} = \frac{3}{32\pi^2} \left\{ \mathcal{M}_{\tilde{t}_i}^4 \left( \log \frac{\mathcal{M}_{\tilde{t}_i}^2}{\Lambda^2} - \frac{3}{2} \right) - 2\mathcal{M}_t^4 \left( \log \frac{\mathcal{M}_t^2}{\Lambda^2} - \frac{3}{2} \right) \right\} \ ,
$$

where $\mathcal{M}_{\tilde{t}_i}^2 (i = 1, \ 2)$ are the field dependent scalar-top quark masses, $\mathcal{M}_t^2$ is the field dependent top quark mass, and $\Lambda$ is the renormalization scale. If the top quark mass and the scalar-top quark mass are identical, then there is no net contribution from the radiative corrections to the tree level Higgs potential.

After spontaneous electroweak symmetry breaking, the masses of the left-handed and the right-handed scalar-top quarks are obtained from the $2 \times 2$ mass matrix for them. They are

$$
\begin{aligned}
m^2_{\tilde{t}_1,\,\tilde{t}_2} &= m_t^2 + \frac{1}{2}(m_Q^2 + m_T^2) + \frac{1}{4} m_Z^2 \cos 2\beta \\
&\mp \left[ \left\{ \frac{1}{2}(m_Q^2 - m_T^2) + \left( \frac{2}{3} m_W^2 - \frac{5}{12} m_Z^2 \right) \cos 2\beta \right\}^2 \right. \\
&\left. + m_t^2 \left( A_t^2 + \lambda^2 x^2 \cot^2 \beta + 2 A_t \lambda x \cot \beta \cos \theta \right) \right]^{\frac{1}{2}} , \qquad (7)
\end{aligned}
$$

where $m_Z^2 = (g_1^2 + g_2^2) v^2/2$ and $m_W^2 = g_2^2 v^2/2$ for $v = \sqrt{v_1^2 + v_2^2} = 175$ GeV.

In the above equation, the scalar-top quark masses contain the mixing term between the left-handed and the right-handed scalar-top quarks as well as the terms proportional to the gauge couplings. If the contributions coming from the $D$-terms in the scalar-top quark mass matrix are neglected, the scalar-top quark masses possess a symmetry under interchange of $m_Q$ and $m_T$. Moreover, one may notice that the scalar-top quark masses possess one CP violation phase $\theta$. If the left-handed and the right-handed scalar-top quarks are degenerate in mass, the mixing term between the scalar-top quarks would vanish and there would be no CP phase in the 1-loop effective potential. The radiative corrections due to the top quark and scalar-top quark contributions do not shift the CP violating vacuum along $\delta$ direction. On the contrary, a shift of the CP violating vacuum along $\theta$ direction is induced by these radiative corrections because the scalar-top quark masses depend on $\theta$.

Now, the two minimum equations

$$
\begin{aligned}
\frac{\partial < V(v_1, v_2, v_3, \theta, \delta) >}{\partial \delta} &= 0 , \\
\frac{\partial < V(v_1, v_2, v_3, \theta, \delta) >}{\partial \theta} &= 0 .
\end{aligned}
$$

at the 1-loop level yield

$$
\begin{aligned}
\tan \delta &= \frac{3 \lambda v^2 \sin 2\beta \sin \theta}{3 \lambda v^2 \sin 2\beta \cos \theta + 2 A_k x} , \\
k &= - \frac{16 \pi^2 v^2 \sin^2 \beta A_\lambda \sin \theta - 3 m_t^2 A_t \sin \theta f(m^2_{\tilde{t}_1},\ m^2_{\tilde{t}_2})}{16 \pi^2 v^2 x \sin^2 \beta \sin(\theta - \delta)} , \qquad (8)
\end{aligned}
$$

where the function $f$ arising from radiative corrections is defined by

$$
f(m_1^2,\ m_2^2) = \frac{1}{(m_2^2 - m_1^2)} \left[ m_1^2 \log \frac{m_1^2}{\Lambda^2} - m_2^2 \log \frac{m_2^2}{\Lambda^2} \right] + 1 .
$$

These are the conditions that are satisfied by the 1-loop CP violating vacuum.

The full mass matrix at 1-loop level for the five neutral Higgs bosons is given by

$$M^2 = M_{ij}^2 + \delta M_{ij}^2$$

where $\delta M_{ij}^2 = \delta M_{ji}^2$ denotes the 1-loop level Higgs boson mass matrix elements, and $M_{ij}^2$ is the tree-level mass matrix obtained in the previous section. We calculate the exact analytical formulae for the elements of the neutral Higgs boson mass matrix at the 1-loop level. Our results are given by the complicated but exact expressions:

$$\begin{aligned}
\delta M_{11}^2 &= \frac{3}{8\pi^2}\left\{\frac{m_t^2\lambda x\Delta_1}{v\sin\beta}+\frac{\cos\beta\Delta}{2v}\right\}^2 \frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2}+\frac{3m_Z^4\cos^2\beta}{128\pi^2v^2}\log\frac{m_{\tilde{t}_2}^2 m_{\tilde{t}_1}^2}{\Lambda^4} \\
&+\frac{3}{16\pi^2v^2}\left\{\frac{2m_t^2\lambda x A_t\cos\theta}{\sin 2\beta}-\left(\frac{4m_W^2}{3}-\frac{5m_Z^2}{6}\right)^2\cos^2\beta\right\} f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) \\
&+\frac{3m_Z^2\cos\beta}{16\pi^2 v}\left\{\frac{m_t^2\lambda x\Delta_1}{v\sin\beta}+\frac{\cos\beta\Delta}{2v}\right\}\frac{\log(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)}\ , \\
\delta M_{22}^2 &= \frac{3}{8\pi^2}\left\{\frac{m_t^2 A_t\Delta_2}{v\sin\beta}-\frac{\sin\beta\Delta}{2v}\right\}^2 \frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2}-\frac{3m_t^4}{4\pi^2v^2\sin^2\beta}\log\frac{m_t^2}{\Lambda^2} \\
&+\frac{3}{16\pi^2v^2}\left\{\frac{m_t^2\lambda x A_t\cot\beta\cos\theta}{\sin^2\beta}-\left(\frac{4m_W^2}{3}-\frac{5m_Z^2}{6}\right)^2\sin^2\beta\right\} f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) \\
&+\frac{3\sin\beta}{16\pi^2 v}\left(\frac{4m_t^2}{\sin^2\beta}-m_Z^2\right)\left\{\frac{m_t^2 A_t\Delta_2}{v\sin\beta}-\frac{\sin\beta\Delta}{2v}\right\}\frac{\log(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)} \\
&+\frac{3}{32\pi^2v^2}\left(\frac{2m_t^2}{\sin\beta}-\frac{m_Z^2\sin\beta}{2}\right)^2\log\frac{m_{\tilde{t}_2}^2 m_{\tilde{t}_1}^2}{\Lambda^4}\ , \\
\delta M_{33}^2 &= \frac{3m_t^4\lambda^2\cot^2\beta}{8\pi^2}\left(\frac{\Delta_1}{m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2}\right)^2 g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) \\
&+\frac{3m_t^2\lambda A_t\cot\beta\cos\theta}{16\pi^2 x} f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)\ , \\
\delta M_{44}^2 &= \frac{3m_t^4\lambda^2x^2A_t^2\sin^2\theta}{8\pi^2v^2\sin^4\beta}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2}+\frac{3m_t^2\lambda x A_t\cos\theta}{16\pi^2v^2\sin^3\beta\cos\beta} f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)\ , \\
\delta M_{55}^2 &= \frac{3m_t^4\lambda^2A_t^2\cot^2\beta\sin^2\theta}{8\pi^2}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2}+\frac{3m_t^2\lambda A_t\cot\beta\cos\theta}{16\pi^2 x} f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)\ , \\
\delta M_{12}^2 &= \frac{3}{8\pi^2}\left\{\frac{m_t^2\lambda x\Delta_1}{v\sin\beta}+\frac{\cos\beta\Delta}{2v}\right\}\left\{\frac{m_t^2 A_t\Delta_2}{v\sin\beta}-\frac{\sin\beta\Delta}{2v}\right\}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2} \\
&+\frac{3}{32\pi^2v^2}\left\{\left(\frac{4m_W^2}{3}-\frac{5m_Z^2}{6}\right)^2\sin 2\beta-\frac{2m_t^2\lambda x A_t\cos\theta}{\sin^2\beta}\right\} f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) \\
&+\frac{3\sin\beta}{32\pi^2 v}\left(\frac{4m_t^2}{\sin^2\beta}-m_Z^2\right)\left\{\frac{m_t^2\lambda x\Delta_1}{v\sin\beta}+\frac{\cos\beta\Delta}{2v}\right\}\frac{\log(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)} \\
&+\frac{3m_Z^2\cos\beta}{32\pi^2 v}\left\{\frac{m_t^2 A_t\Delta_2}{v\sin\beta}-\frac{\sin\beta\Delta}{2v}\right\}\frac{\log(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{3m_Z^2\sin 2\beta}{256\pi^2 v^2}\left(\frac{4m_t^2}{\sin^2\beta}-m_Z^2\right)\log\frac{m_{\tilde{t}_2}^2 m_{\tilde{t}_1}^2}{\Lambda^4}\ , \\
\delta M_{13}^2 &= \frac{3}{8\pi^2}\left\{\frac{m_t^2\lambda x\Delta_1}{v\sin\beta}+\frac{\cos\beta\Delta}{2v}\right\}\frac{m_t^2\lambda\cot\beta\Delta_1}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2}g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) \\
&-\frac{3m_t^2\lambda}{16\pi^2 v\sin\beta}(A_t\cos\theta+2\lambda x\cot\beta)f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) \\
&+\frac{3m_Z^2 m_t^2\lambda\cos\beta\cot\beta}{32\pi^2 v}\left(\frac{\Delta_1}{m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2}\right)\log\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\ , \\
\delta M_{14}^2 &= -\frac{3m_t^2\lambda x A_t\sin\theta}{8\pi^2 v\sin^2\beta}\left\{\frac{m_t^2\lambda x\Delta_1}{v\sin\beta}+\frac{\cos\beta\Delta}{2v}\right\}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2} \\
&+\frac{3m_t^2\lambda x A_t\cot\beta\sin\theta}{16\pi^2 v^2\sin\beta}\left\{f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)-\frac{m_Z^2}{2(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)}\log\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\right\}\ , \\
\delta M_{15}^2 &= -\frac{3m_t^2\lambda A_t\cot\beta\sin\theta}{8\pi^2}\left\{\frac{m_t^2\lambda x\Delta_1}{v\sin\beta}+\frac{\cos\beta\Delta}{2v}\right\}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2} \\
&+\frac{3m_t^2\lambda A_t\cot\beta\sin\theta}{16\pi^2 v\sin\beta}\left\{f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)-\frac{m_Z^2}{2(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)}\log\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\right\}\ , \\
\delta M_{23}^2 &= \frac{3}{8\pi^2}\left\{\frac{m_t^2 A_t\Delta_2}{v\sin\beta}-\frac{\sin\beta\Delta}{2v}\right\}\frac{m_t^2\lambda\cot\beta\Delta_1}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2}g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) \\
&-\frac{3m_t^2\lambda A_t\cot\beta\cos\theta}{16\pi^2 v\sin\beta}f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) \\
&+\frac{3m_t^2\lambda\cos\beta}{32\pi^2 v}\left(\frac{4m_t^2}{\sin^2\beta}-m_Z^2\right)\left(\frac{\Delta_1}{m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2}\right)\log\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}, \\
\delta M_{24}^2 &= -\frac{3m_t^2\lambda x A_t\sin\theta}{8\pi^2 v\sin^2\beta}\left\{\frac{m_t^2 A_t\Delta_2}{v\sin\beta}-\frac{\sin\beta\Delta}{2v}\right\}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2} \\
&+\frac{3m_t^2\lambda x A_t\sin\theta}{16\pi^2 v^2\sin\beta}\left\{f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)+\frac{m_Z^2}{2(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)}\log\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\right\}\ , \\
\delta M_{25}^2 &= -\frac{3m_t^2\lambda A_t\cot\beta\sin\theta}{8\pi^2}\left\{\frac{m_t^2 A_t\Delta_2}{v\sin\beta}-\frac{\sin\beta\Delta}{2v}\right\}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2} \\
&+\frac{3m_t^2\lambda A_t\cot\beta\sin\theta}{16\pi^2 v\sin\beta}\left\{f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)+\frac{m_Z^2}{2(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)}\log\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\right\}\ , \\
\delta M_{34}^2 &= -\frac{3m_t^4\lambda^2 x A_t\cot\beta\sin\theta}{8\pi^2 v\sin^2\beta}\frac{\Delta_1 g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2}+\frac{3m_t^2\lambda A_t\sin\theta}{16\pi^2 v\sin^2\beta}f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)\ , \\
\delta M_{35}^2 &= -\frac{3m_t^4\lambda^2 A_t\cot^2\beta\sin\theta}{8\pi^2}\frac{\Delta_1 g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2}\ , \\
\delta M_{45}^2 &= \frac{3m_t^4\lambda^2 x A_t^2\cot\beta\sin^2\theta}{8\pi^2 v\sin^2\beta}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2}+\frac{3m_t^2\lambda A_t\cos\theta}{16\pi^2 v\sin^2\beta}f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)\ ,
\end{aligned}
\tag{9}
$$

with

$$\begin{aligned}
\Delta_1 &= A_t \cos\theta + \lambda x \cot\beta \ , \\
\Delta_2 &= A_t + \lambda x \cot\beta \cos\theta \ , \\
\Delta &= \left\{ (m_Q^2 - m_T^2) + \left( \frac{4m_W^2}{3} - \frac{5m_Z^2}{6} \right) \cos 2\beta \right\} \left( \frac{4m_W^2}{3} - \frac{5m_Z^2}{6} \right) \ ,
\end{aligned} \tag{10}$$

and

$$g(m_1^2, m_2^2) = \frac{m_2^2 + m_1^2}{m_1^2 - m_2^2} \log \frac{m_2^2}{m_1^2} + 2 \ .$$

Here, in the limit of $\sin\theta = 0$, some of the elements of $\delta M^2$ vanish. In this limit, the radiatively-corrected mass matrix of the neutral Higgs bosons reduces to the one obtained without the spontaneous violation of the CP symmetry [7].

We also remark that the spontaneous violation of the CP symmetry forbids a non-zero CP phase in our scenario at the 1-loop level if $\theta = \delta$ at the tree level. This is exactly what the Georgi-Pais theorem [13] says: The radiative CP violation can be realized when at the tree level there exist massless Higgs bosons other than Goldstone bosons. That is, spontaneous CP violation does not occur in the NMSSM Higgs sector through only radiative corrections because at the tree level there is no pseudo-Goldstone boson.

One can notice that the scalar-pseudoscalar mixing elements of the radiatively corrected mass matrix $\delta M^2$ of the neutral Higgs bosons are nonzero, as far as $\theta$ is nonzero, assuming the non-degeneracy of the left-handed and the right-handed scalar-top quark masses. The magnitudes of these elements are proportion to $\sin\theta$. The scalar-pseudoscalar mixings between two Higgs doublets generated by the radiative corrections would not occur if the degenracy of the left-handed and the right-handed scalar-top quark masses in the 1-loop effective potential is assumed.

The five physical neutral Higgs bosons are defined as the mass eigenstates, obtained by diagonalizing the mass matrix at 1-loop level, by the help of an orthogonal transformation matrix. The elements of this orthogonal transformation matrix determine the couplings of the physical neutral Higgs bosons to the other states in the model. Let us denote the physical five neutral Higgs bosons as $h_i$ ($i$ = 1,2,3,4,5). We take the mass eigenvalues in increasing order of the mass eigenstates $h_i$.

Constraints of the NMSSM parameter space arise from searches for the Higgs bosons at the LEP1. In our numerical analysis, we have used the following experimental constraints from LEP1. The fact that two Higgs bosons have not been produced in the decay of $Z$ gives the condition of $m_{h_1} + m_{h_2} > m_Z$. In the case of $m_{h_1} + m_{h_2} < m_Z$, the decay $Z \to h_1 h_2$ is kinematically allowed and the branching ratio $B(Z \to h_1 h_2)$ should be smaller than $10^{-7}$. For two Higgs bosons ($h_1$, $h_2$), both $B(Z \to h_1 l^+ l^-)$ and $B(Z \to h_2 l^+ l^-)$ should be smaller than $1.3 \times 10^{-7}$. In our numerical analyses, the renormalization scale and the mass of the top-quark [14] is fixed as 1000 GeV and 175 GeV, respectively. The upper bounds on $\lambda$ and $k$ are given as 0.87 and 0.63, respectively, by the renormalization group analysis of the NMSSM [15]. The phase $\delta$ and coupling costant $k$ is determined by two minimum conditions (Eq. 8) of the Higgs potential. Assuming the same soft SUSY breaking scalar-quark masses ($m_Q = m_T$), we can determine they by the relation,

$\Delta m_{\tilde{t}}^2/m_{\tilde{t}}^2 \sim 1/30$. Here the scalar-top quark masses are constrained to be very heavy for later convenience.

Within the context of our model, we plot the masses of the neutral Higgs bosons by the Monte Carlo method using the above formulae for reasonable regions of the parameter space. Fig. 1 shows at the 1-loop level the three lighter neutral Higgs boson masses $m_{h_1}$, $m_{h_2}$, and $m_{h_3}$ as functions of $A_t$, for $0 < \theta < 2\pi$, $1 < \tan\beta < 10$, $0 < \lambda < 0.87$, and $0 < A_\lambda, -A_k, x < 1000$ GeV. One can see in Fig. 1a that many points are excluded for the range of $m_{h_1} < m_Z$ by the negative experimental results at LEP1; but the LEP1 data do not completely exclude the existence of a massless neutral Higgs boson of the model at the 1-loop level.

Our numerical result does not seem to be compatible with that of recent research [10]. It was recently pointed out that at the 1-loop level a wide region of the NMSSM parameter space is excluded for spontaneous CP violation. Especially the range of $\tan\beta > 1$ is completely ruled out in this scenario and then in this analysis $m_{h_1}$ is relatively small ( $\sim 35$ GeV) for most of the parameter space. Here Haba at al. [10] did not consider the range of $A_k < 0$ in their analysis. The sign of $A_k$ values need not be positive when CP is spontaneously violated in the Higgs sector.

The range of $A_k < 0$ plays a important role in allowing spontaneous CP violation in the NMSSM because of in the minimum equation the sign of $k$ is negative for a reasonable parameter space. Thus the range of $\tan\beta > 1$ is allowed for spontaneous CP violation in the Higgs sector of the NMSSM in our analysis. In the parameter space of Fig. 1 but the range of $A_k > 0$, spontaneous CP violation is not allowded in the NMSSM even though radiative corrections are included. Figs. 1b, 1c display the allowed ranges of $60 < m_{h_2} < 245$ GeV for $m_{h_2}$ and of $100 < m_{h_2} < 525$ GeV for $m_{h_3}$. In these figures the upper and lower bounds on the neutral Higgs mass boson masses are theoretical and experimental bounds, respectively. We also calculated $m_{h_1}$ for the same parameter space as that of Fig. 1 without spontaneous CP violation in the NMSSM. The upper bound on $m_{h_1}$ is decreased by spontaneous CP violation in the Higgs sector.

# IV. $K$-$\bar{K}$ MIXING IN NMSSM

We now explore their phenomenological implications for spontaneous CP-violating effects in $K$-$\bar{K}$ mixing. In the SM, the $K$-$\bar{K}$ mixing arises from the $W$-exchange box diagram giving rise to the $\Delta S = 2$ operator at the 1-loop level. In the NMSSM, there are dominant sources for CP violation in the $K$-$\bar{K}$ system through box diagrams involving superpartners (superbox). In the superbox diagrams, the gauge fermion couplings are described by the super-CKM unitary matrices which diagonalize the scalar quark mass matrix [16]. The super-CKM matrix is real in the spontaneous CP violation scenario that we concentrate on.

After electroweak symmetry breaking takes place, the gauginos and Higgsinos with the same spin combine to form mass eigenstates, charginos and neutralinos. The complex mixing between gauginos and Higgsinos induce a complex chargino or neutralino propagator [17]. In the case, the superbox diagrams contain couplings among quark scalar-quark, and Higgsino so that their contribution to the imaginary part of $K$-$\bar{K}$ mixing is suppressed by a factor of $m_q/m_W$. If the superbox diagrams contain the weak phase in a scalar quark propagator, the superbox digrams receive a suppression factor of $m_q/m_{\tilde{q}}$. Thus, the dominant contributions to the imaginary part of $K$-$\bar{K}$ mixing come from those including the left-right scalar-top quark mixing. The superbox

diagrams for these contributions are displayed in other papers [17, 18]. Assuming that scalar-top quarks are very heavy $m_{\tilde{t}} \gg m_{\tilde{W}}$ and the upper bound on $\Delta m_{\tilde{t}}^2/m_{\tilde{t}}^2$ is saturated as 1/30, the ratio of the imaginary and real parts of the $K$-$\bar{K}$ mixing can be described by the quantity [18]

$$3\left(\frac{m_t}{v\sin\beta}\right)^2 \frac{v m_{\rm LR}^2 V_{13} z \sin\phi}{m_{\tilde{W}} m_{\tilde{t}}^2} \left(\frac{\Delta m_{\tilde{t}}^2}{m_{\tilde{t}}^2}\right)^{-1} , \tag{11}$$

where $z = 0.5$ is a partial cancellation factor, $m_{\rm LR}$ is the left-right scalar-top quark mixing, $m_{\tilde{t}}$ is the average of the scalar-top quark masses, $V_{13} = 10^{-2}$ is a element of the super-CKM matrix for the scalar-quark, and $m_{\tilde{W}}$ is the superpartner (wino) of SU(2). The CP-violating phase $\phi$ is expressed as a function of two phases $(\theta,\ \delta)$ of VEV's.

We impose the experimental constraints at LEP1 on the NMSSM parameter space and then estimate numerically CP-violating effects in $K$-$\bar{K}$ mixing by using the parameters obtained in the context of spontaneous CP-violating scenario. The experimental upper bound on the ratio of $K$-$\bar{K}$ mixing is given by $6 \times 10^{-3}$ [16]. This bound impose a constraint on the weak CP phase. The wino is not exist as a mass eigenstate. Thus we fix as $m_{\tilde{W}} = 100$ GeV for heavy scalar-top quarks. We use Eq. 11 for the ratio of the $K-\bar{K}$ mixing and plot in Fig. 2a the lower bound on the weak phase $\phi$ as a function of $A_t$, for $0 < \tan\beta < 10$, $0 < \theta < 2\pi$, $0 < \lambda < 0.87$, and $0 < A_\lambda, -A_k, x <$ 1000 GeV. One can notice that the lower bound on $\phi$ becomes small as the value of $A_t$ decrease. Fig. 2b shows the lower bound on the weak phase $\phi$ as the same parameter space of as that of Fig. 1a but as a function of $\tan\beta$. In Fig. 2b the lower bound on $\phi$ increases as the values of $\tan\beta$ decrease.

## V. CONCLUSIONS

Spontaneous CP violation is investigated in the Higgs sector of the NMSSM. In previous analyses [9], it has been shown that the spontaneous violation of the CP symmetry can be realized in the NMSSM Higgs sector by radiative corrections coming from the degenerate scalar-top quark masses. In this case, however, the presence of the CP violating vacuum requires a very light Higgs boson in the model. By considering the negative results in the Higgs search at LEP1, the possibility of the spontaneous violation of the CP symmetry is almost completely excluded in the Higgs sector of the NMSSM, if the scalar-top quark masses are degenerate. The numerical analysis also have shown [10] that the situation is hardly improved when the radiative corrections including the mass spliting effect between the scalar-top quarks are taken into account.

Here we reanalyze the possibility of spontaneous CP violation in the Higgs sector of the NMSSM with the non-degeneracy of the scalar-top quark masses in the 1-loop effective potential. In this analysis the Higgs potential contains radiative corrections due to the top-quark and scalar-top quark contributions. In the NMSSM the mass matrix of the neutral Higgs boson is analytically derived in spontaneous CP violation scenario. In the range of $A_k > 0$ CP violation is not realized even though radiative corrections due to the these contributions are included in the Higgs sector. On the contrary, CP violation scenario is viable in the range of $A_k < 0$. Assuming both scalar-top quarks are much heavier than the wino and scalar-top quarks are maximally degenerated in their masses, the neutral Higgs boson masses are numerically calculated. The three neutral Higgs bosons

are relatively light. The upper bound on the lightest neutral Higgs boson mass is about 140 GeV under our choice for the parameter space. This upper bound increases if spontaneous CP violation does not occur in the Higgs sector. In the 1-loop parameter space of the NMSSM, the lower bound on the phase $\phi$ is calculated by using the experimental result for the ratio of $K$-$\bar{K}$ mixing. The lower bound on the phase $\phi$ increase as $\tan\beta$ values decrease or as $A_t$ values increase. In the NMSSM Higgs sector, a possibility of spontaneous CP violation scenario can not be completely ruled out by the Higgs search at LEP.

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## Figure Captions

Fig. 1 : (a) $m_{h_1}$, (b) $m_{h_2}$, and (c) $m_{h_3}$ as a function of $A_t$, for $0 < \theta < 2\pi$, $1 < \tan\beta < 10$, $0 < \lambda < 0.87$, and $0 < A_\lambda, -A_k, x < 1000$ GeV.

Fig. 2 : The lower bound on $\phi$ as a function of (a) $A_t$ and (b) $\tan\beta$

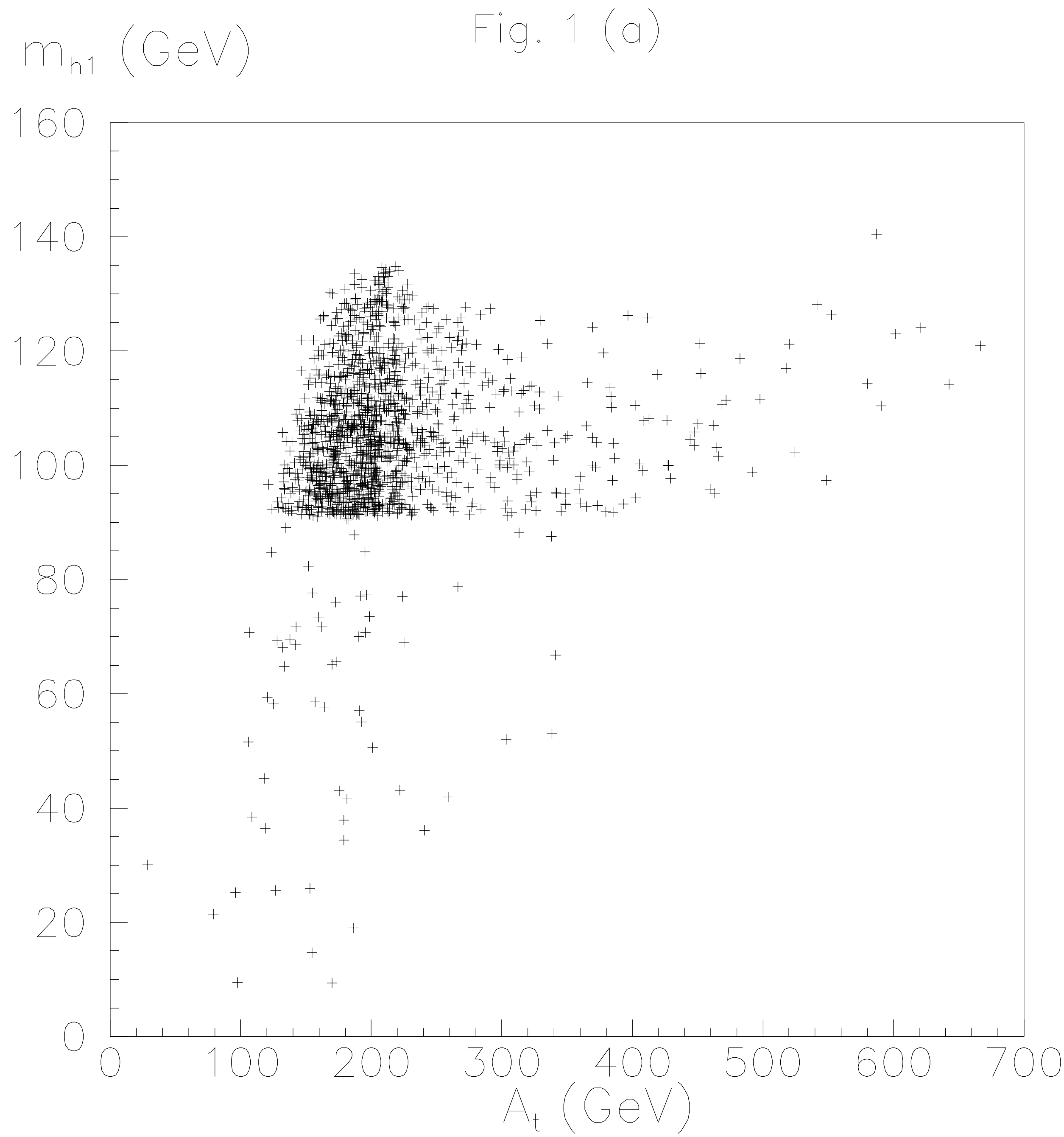


FIGURE 1: fig1a

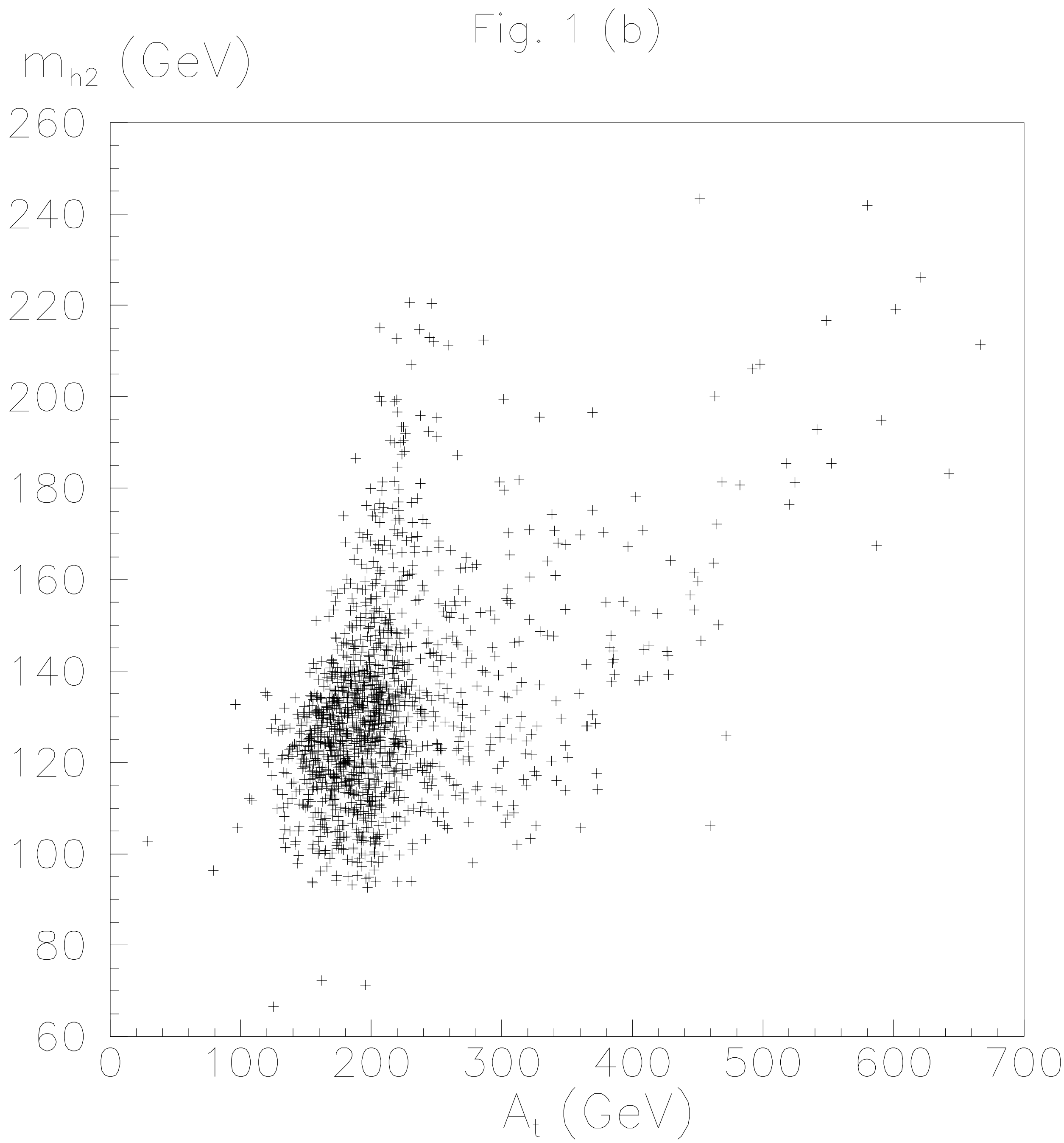


FIGURE 2: fig1b

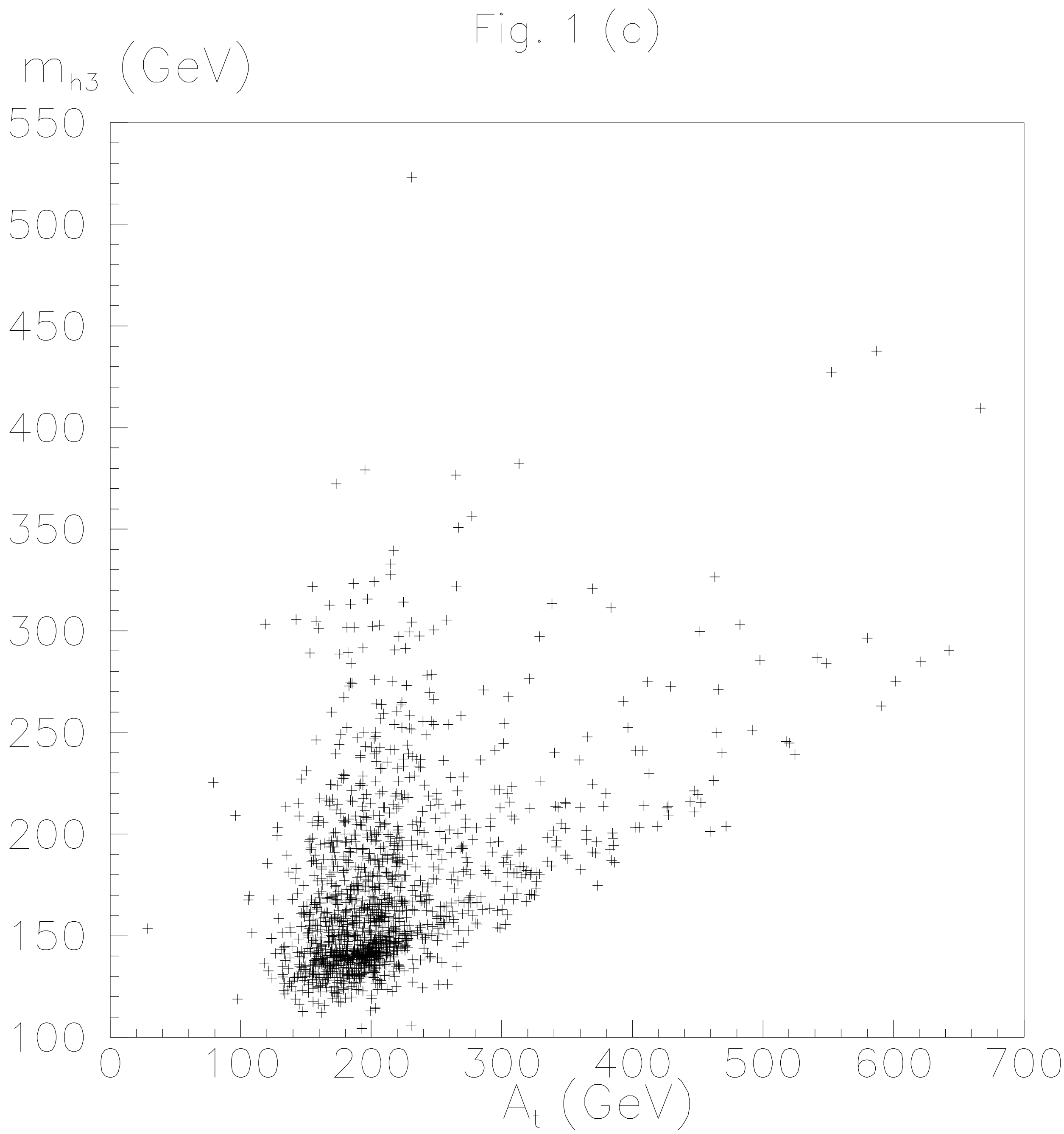


FIGURE 3: fig1c

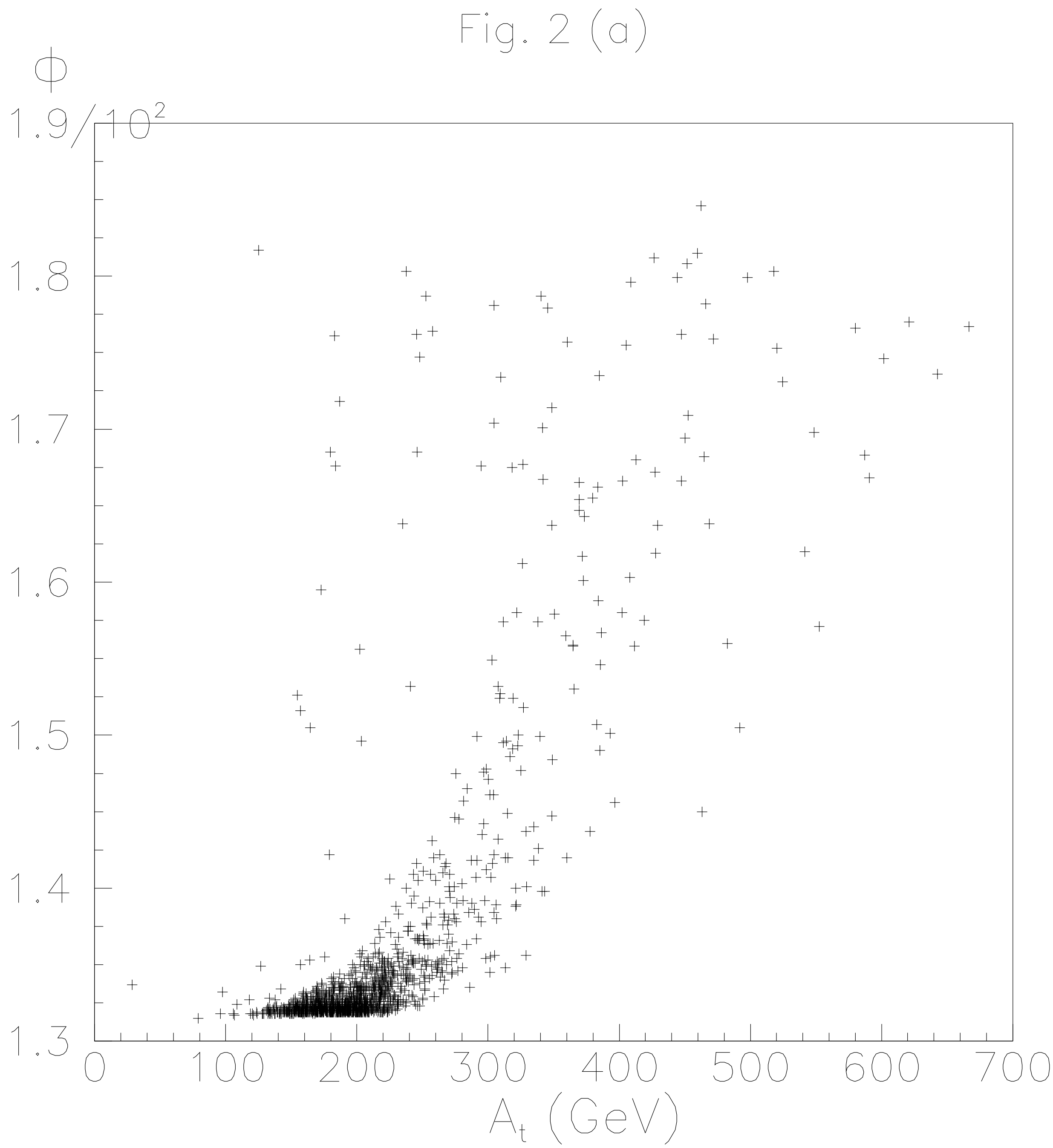


FIGURE 4: fig2a

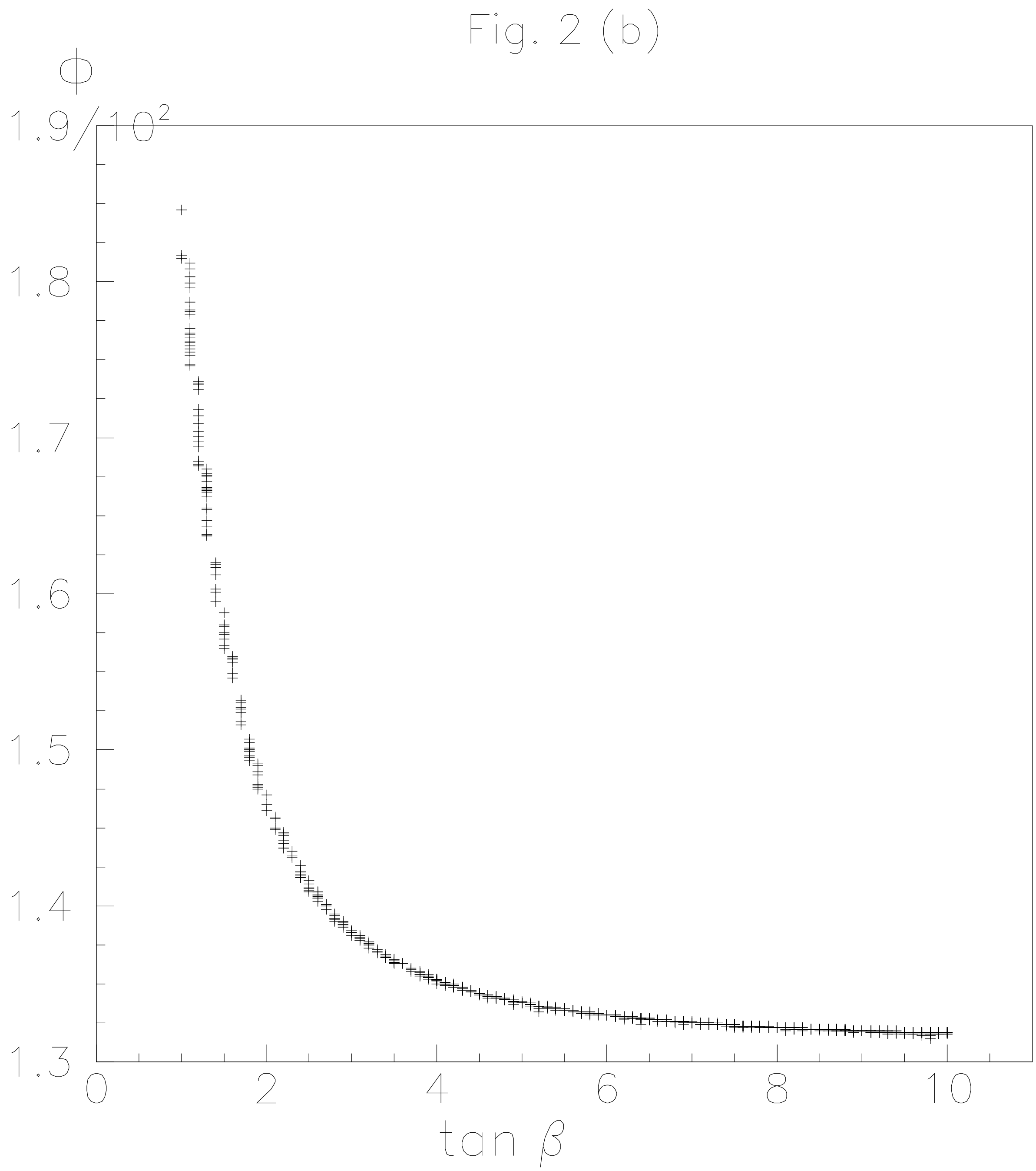


FIGURE 5: fig2b